CrashSim: An Efficient Algorithm for Computing SimRank over Static and Temporal Graphs

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- Motivation

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- Experimental Evaluation
- Conclusion

Background

- SimRank Overview
- Motivation

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Background

- Similarity assessment plays a vital role in our lives.

Recommender System

Citation Graph

Collaboration Network

Background

- SimRank
  - Node-to-node measurement based on the topology of graphs (KDD'02)
  - Basic assumption
    - *Two nodes will be similar if they are both highly relevant to similar nodes*

- Two Forms
  - Original definition (KDD'02)

$$s(u,v) = \begin{cases} 1, & \text{if } u = v \\ \dfrac{c}{|I(u)| \cdot |I(v)|} \sum\limits_{x \in I(u), y \in I(v)} s(x,y), & \text{otherwise.} \end{cases}$$

  - √c- walk (SIGMOD'16)

$$s(u,v) = \Pr\left[W(u) \text{ and } W(v) \text{ meet}\right]$$

Background

- Temporal Graph

- Temporal SimRank queries: *threshold* and *trend*

Problem Definition

- Preliminaries
- Problem Definition

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Preliminaries

Definition 1 ($\sqrt{c}$-walk). *Let c denotes the decay factor in the definition of SimRank, a $\sqrt{c}$-walk in G is defined such that:*

- *In each step of the random walk, we have $1 - \sqrt{c}$ probability to stop.*
- *For the remaining $\sqrt{c}$ probability, one of the in-neighbors of the current node is selected uniformly at random as the next step.*

$$
\begin{aligned}
s(u, v) &= \Pr\left[W(u) \text{ and } W(v) \text{ meet}\right] \\
&= \sum_i \Pr\left[W(u) \text{ and } W(v) \text{ first meet at } u_i\right].
\end{aligned}
$$

SLING algorithm (SIGMOD'16)

ProbeSim algorithm (VLDB'17)

Problem Definition

Problem (Temporal SimRank Queries)
Given: $G, \ u, \ [T_1, T_t]$
Return: node set Ω, such that the SimRank of u and each node $v \in \Omega$ continuously meet a certain query requirement during the entire query interval $[T_1, T_t]$

Problem (Temporal SimRank Trend Query)
Given: $G, \ u, \ [T_1, T_t]$
Return: node set Ω, such that the SimRank of u and each node $v \in \Omega$ is continuously **increasing (or decreasing)** during the entire query interval $[T_1, T_t]$

Problem (Temporal SimRank Threshold Query)
Given: $G, \ u, \ [T_1, T_t], \theta$
Return: node set Ω, such that the SimRank of u and each node $v \in \Omega$ is **greater than** $\boldsymbol{\theta}$ during the entire query interval $[T_1, T_t]$

Our Approach

- CrashSim Algorithm -— static graphs
- CrashSim-T Algorithm — temporal graphs

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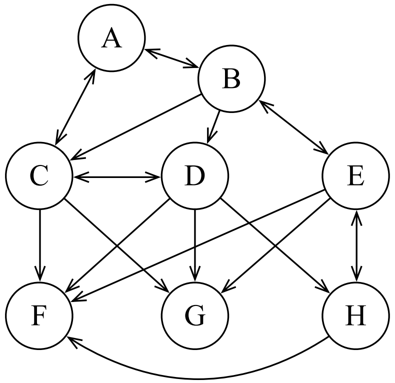
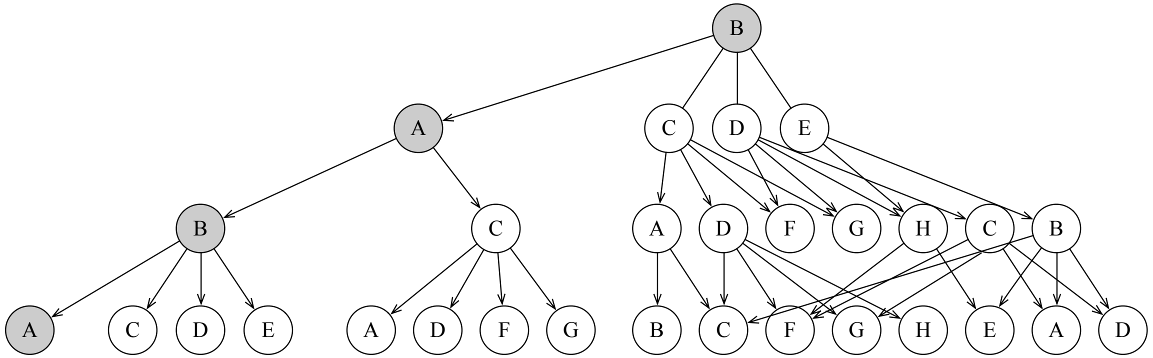
CrashSim Algorithm

- Motivation
  - ProbeSim (VLDB'17) is the state-of-the-art algorithm for SimRak computation over static graph

$$s(u,v) = \Pr[W(u) \text{ and } W(v) \text{ meet}]$$
$$= \sum_i \Pr[W(u) \text{ and } W(v) \text{ first meet at } u_i].$$





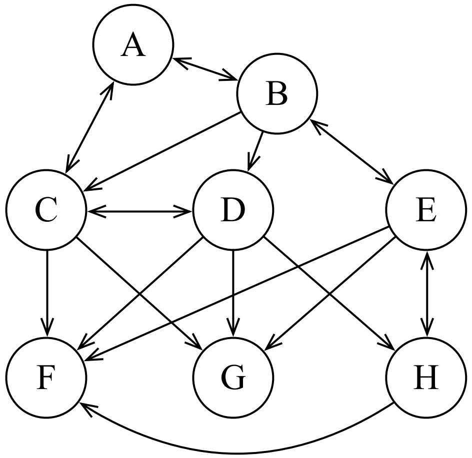
- Drawbacks
  - *redundant computations*
  - *the length of √c-walk determine the computation costs*

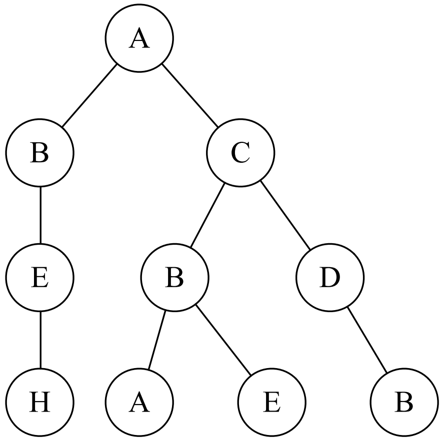
CrashSim Algorithm

- Key idea
  - *Constrain the length of $\sqrt{c}$-walk to l_{max}*
  - *A reverse reachable tree of source u with the limited length of $\sqrt{c}$-walk, l_{max}*
  - *Still obtain SimRank estimators with the same guaranteed error bound of the ProbeSim*

Problem (Approximation Guarantee)
Given: $G, \ u, \ \varepsilon, \ \delta$
Return: $s(u, v)$ such that $|s(u, v) \ - sim(u, v)| \leq \varepsilon$ with at least $1 - \delta$ probability

CrashSim Algorithm





$U(0, A) = 1$

$U(1, B) = U(0, A) \times \frac{\sqrt{c}}{|I(B)|} = 1 \times \frac{0.5}{2} = 0.25$

$U(1, C) = U(0, A) \times \frac{\sqrt{c}}{|I(C)|} = 1 \times \frac{0.5}{3} = 0.167$

$U(2, E) = 0.0625$, $U(2, B) = 0.0417$, $U(2, D) = 0.0417$

$U(3, H) = 0.0156$, $U(3, A) = 0.0104$
$U(3, E) = 0.0104$, $U(3, B) = 0.0104$

In the k-th trial, $W(C) = (C, D, B, A)$

$$s_k(\mathrm{A}, \mathrm{C}) = U(0, \mathrm{C}) + U(1, \mathrm{D}) + U(2, \mathrm{B}) + U(3, \mathrm{A})$$
$$= 0 + 0 + 0.0417 + 0.0104 = 0.0521$$

CrashSim Algorithm

Theorem 1. *For any node* $v \in \Omega$, $sim(u,v)$ *and its estimator* $s(u,v)$ *satisfies* $\Pr\{|sim(u,v) - s(u,v)| \leq \varepsilon\} \geq 1 - \delta$, *where* $s(u,v) = \frac{1}{n_r}\sum_{k=1}^{n_r}\sum_{i=2}^{\min(l,l_{\max})} P(v, W(u,i))$, $l_{\max} = \frac{1+\sqrt{c}}{(1-\sqrt{c})^2}$, $n_r = \frac{3c}{(\varepsilon - p\varepsilon_t)^2}\log\frac{n}{\delta}$, $\varepsilon_t = (\sqrt{c})^{l_{\max}}$, $p = \sum_{k=1}^{l_{\max}} (\sqrt{c})^{k-1}(1-\sqrt{c})$.

Time Complexity: $O(m + n_r \cdot |\Omega|) = O(m + \frac{3c}{(\varepsilon - p\varepsilon_t)^2}\log\frac{n}{\delta} \cdot |\Omega|)$.

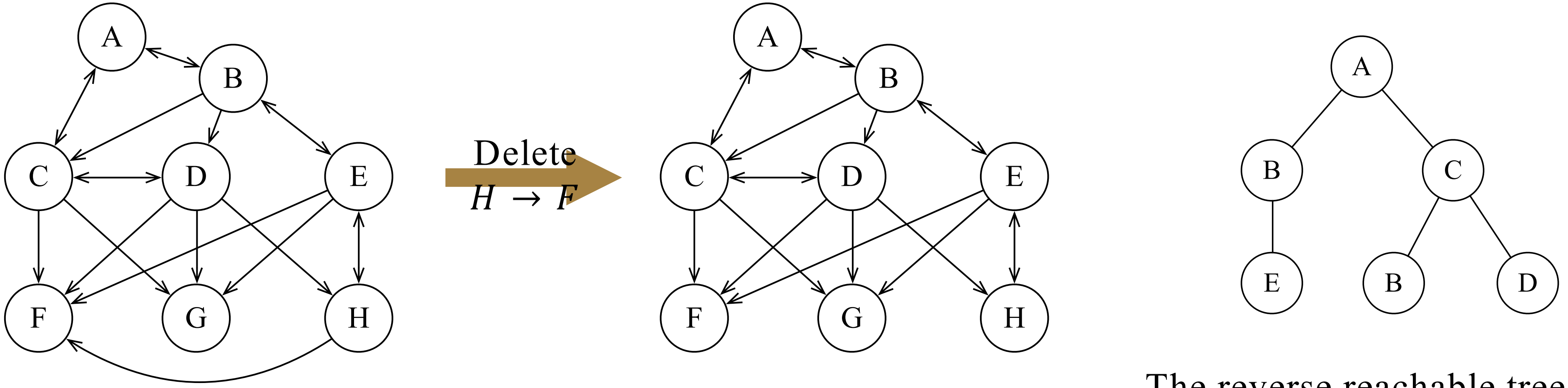
CrashSim-T Algorithm

- Two opportunities
  - *Unnecessary to compute the SimRank between u and the candidate node set Ω at each time instant*
  - *The size of node set Ω can only gradually reduce over time*

- CrashSim naturally supports the computation of SimRank of the source u and a partial set of nodes.

CrashSim-T Algorithm --- Delta Pruning

- Affected area of a changed edge $x \rightarrow y$
  - *The altered nodes in the reverse reachable tree of u*
  - $l_{max} - 1$ *length reachable nodes of y*
- Delta pruning: ignore the nodes of an unaffected area



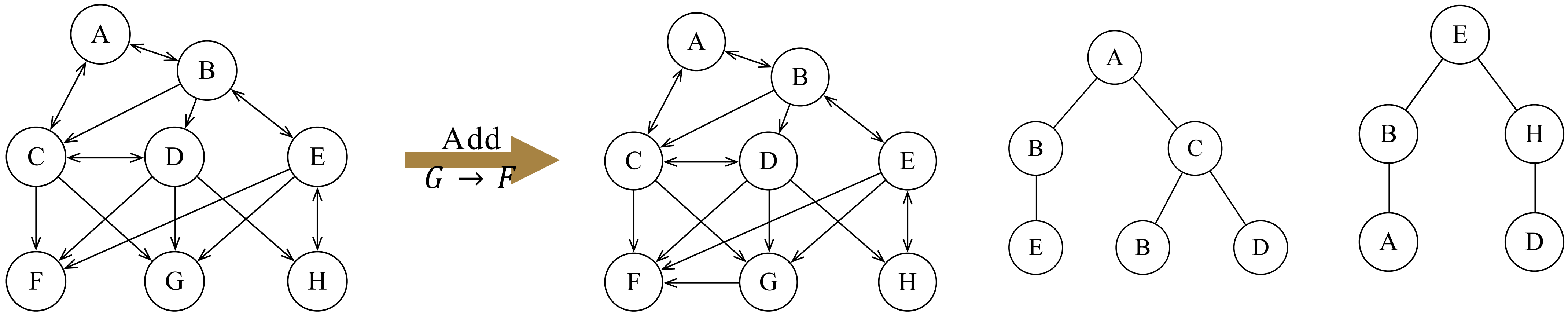
The reverse reachable tree of A remains unchanged.

CrashSim-T Algorithm --- Difference Pruning

- Related area: the l_{max} length reverse reachable tree of u and v

$$s(u,v) = Pr[W(u)\ and\ W(v)\ meet]$$
$$= \sum_i Pr[W(u)\ and\ W(v)\ first\ meet\ at\ u_i].$$

- Difference pruning: filter out those nodes whose related area is unchanged



The reverse reachable tree of A and E remains unchanged.

CrashSim-T Algorithm

- Main idea
  - *Check whether the conditions of delta and difference pruning are satisfied*
  - *If so, disregard those nodes as part of the candidate node set*
  - *Invoke CrashSim algorithm to compute u and residual nodes*
  - *According to different query requirements to filter out unsatisfied nodes*

Experiments and Conclusion

- Experimental Evaluation
- Conclusion

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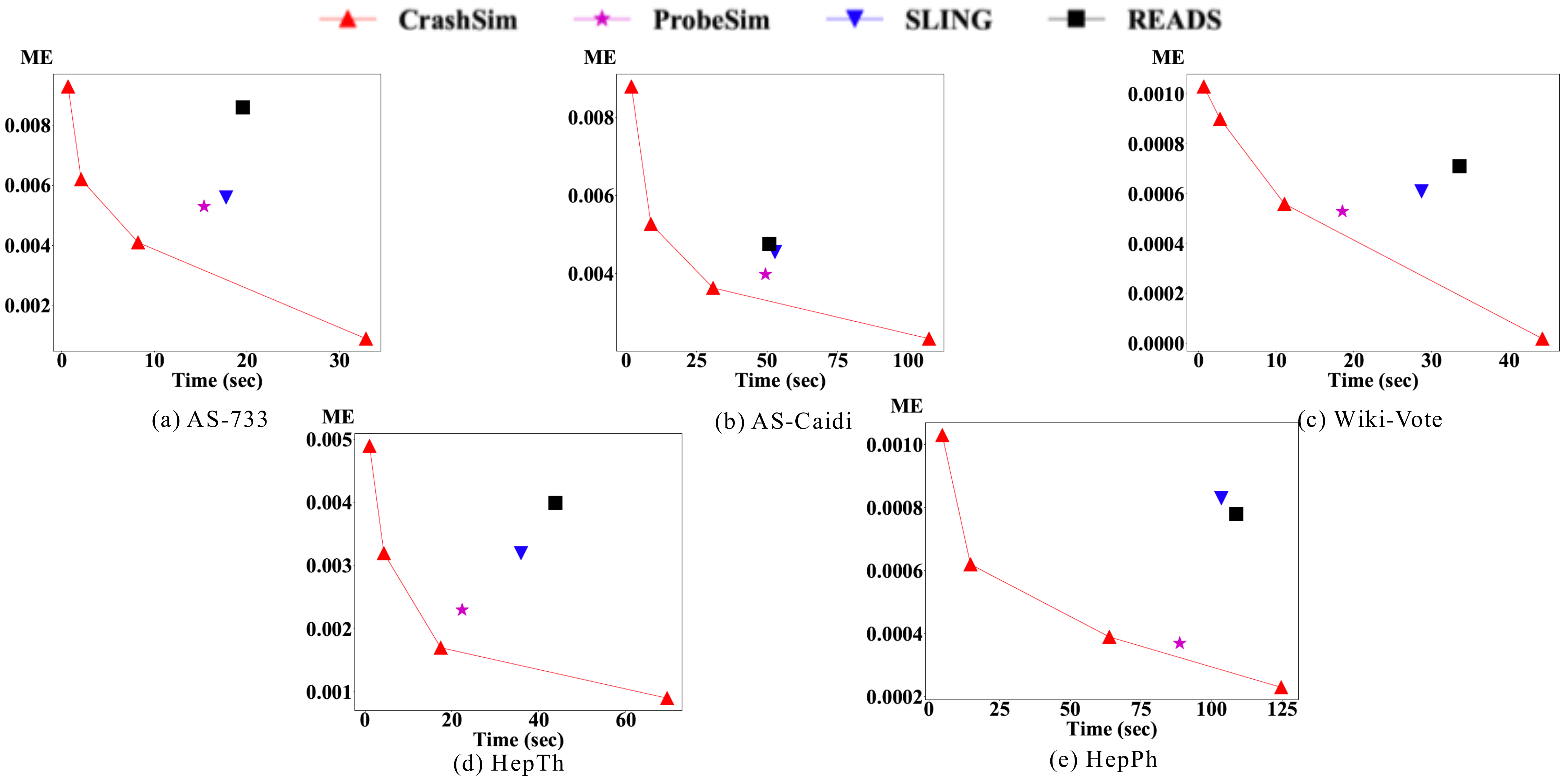
Experimental Evaluation

- Datasets

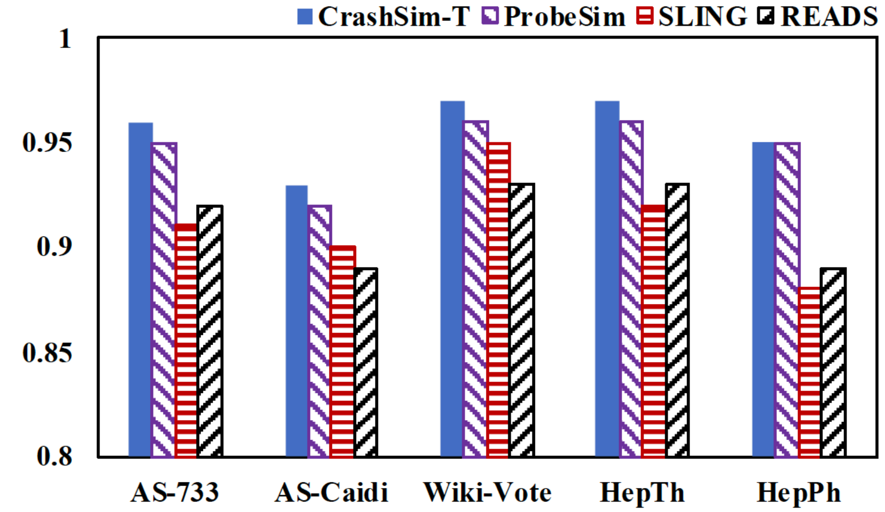
Datasets	Type	n	m	t
AS-733	Undirected	6,474	13,233	733
AS-Caidi	Directed	26,475	106,762	122
Wiki-Vote	Directed	7,155	103,689	100
HepTh	Undirected	9,877	25,998	100
HepPh	Directed	34,546	421,578	100

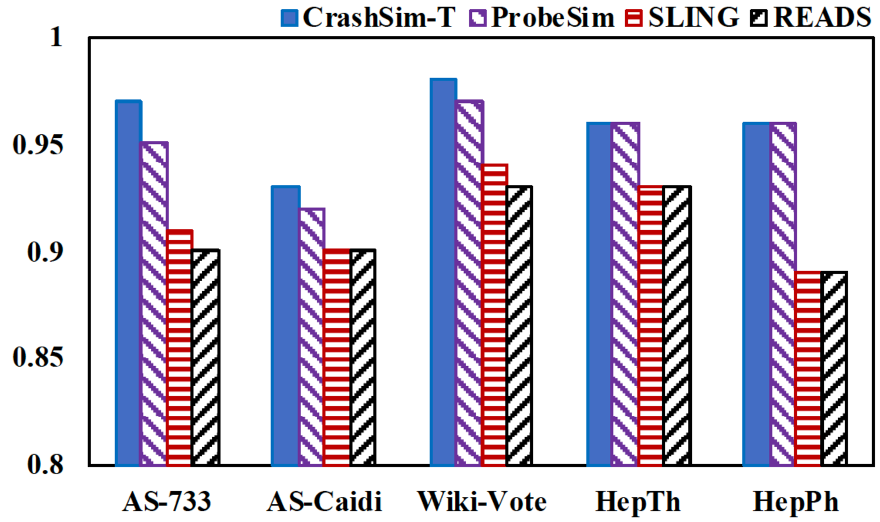
- Comparison baselines
  - SLING (SIGMOD'16), ProbeSim (VLDB'17), READS (VLDB'17)
- Setting and metrics
  - ε varies from 0.0125, 0.025, 0.05 to 0.1
  - $ME = \max|s(u,v) - sim(u,v)|\ (v \in V)$
  - $precision = \frac{v(k_1) \cap v(k_2)}{\max(k_1, k_2)}$

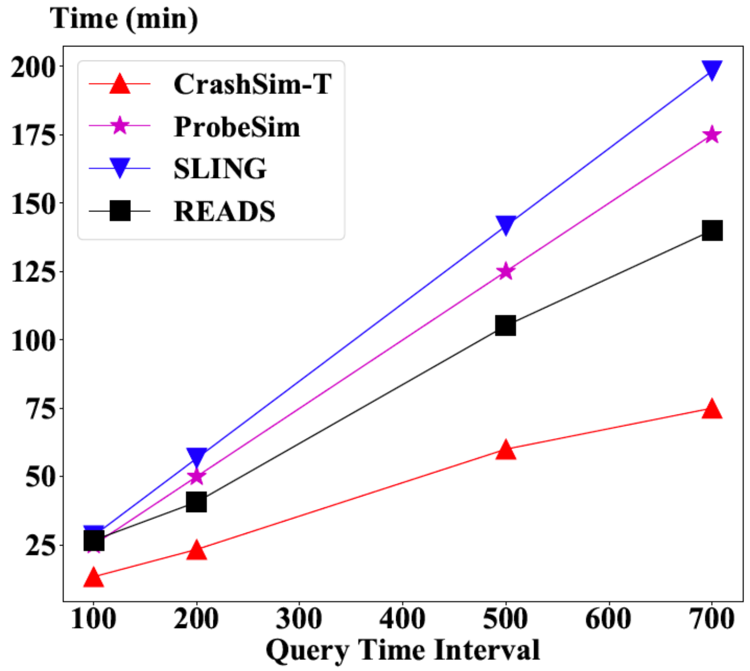
Experimental Evaluation



Experimental Evaluation







The impact of the query interval on the response time of the algorithms

Conclusion

- Propose **CrashSim** algorithm, an index-free algorithm for single-source and partial SimRank computation in static graphs
- Introduce **CrashSim-T** --- an extension to CrashSim to solve SimRank queries over temporal graphs
- Experiments show that both CrashSim and CrashSim-T outperform the state-of-the-art algorithms.

THANKS

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